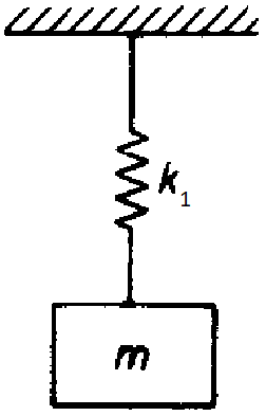
Problem 2.2

A spring-mass system, k_1 and m, has a natural frequency of f_1. If a second spring k_2 is added in series with the first spring, the natural frequency is lowered to $\frac{1}{2}f_1$. Determine k_2 in terms of k_1.

Solution

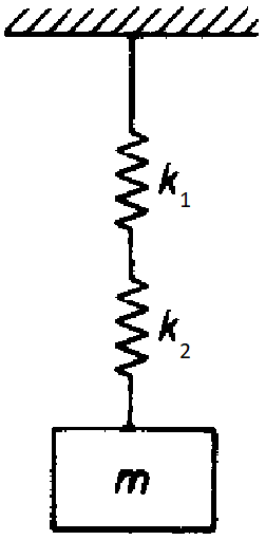
Below is the initial spring-mass system with spring stiffness k_1 and mass m.



The natural frequency for this system is

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}}.$$

Now we add a second spring to the system in series.



These two springs in series can be replaced by one effective spring stiffness,

$$k_{\text{eff}} = \frac{1}{\frac{1}{k_1}+\frac{1}{k_2}} = \frac{k_1 k_2}{k_1 + k_2}.$$

The natural frequency for this system is

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}}$$
$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1+k_2)}}$$
$$\frac{1}{2}f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1+k_2)}}$$
$$\frac{1}{2}\left(\frac{1}{2\pi}\sqrt{\frac{k_1}{m}}\right) = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1+k_2)}}$$
$$\frac{1}{2} = \sqrt{\frac{k_2}{k_1+k_2}}$$
$$\frac{1}{4} = \frac{k_2}{k_1+k_2}$$
$$k_1 + k_2 = 4k_2$$
$$k_1 = 3k_2.$$

Therefore,

$$k_2 = \frac{1}{3}k_1.$$